

Introduction

Contents of Manual

This manual contains the materials needed to teach the partial products algorithm for multiplication using the Concrete-Representational-Abstract (C-R-A) method of instruction with an emphasis on the mathematical practices infused throughout the Numbers and Operations standards in most states. State standards call for computation using strategies based on place value. One such strategy is the partial products algorithm. The materials allow for computation instruction within the context of meaningful problem situations. As students master and demonstrate understanding of multiplication, the materials assist them in understanding its relation to other operations.

Students Who Need This Manual

Students who consistently solve problems as shown below may need this program.

$$\begin{array}{r} 2 \\ 2 \ 4 \\ \times 3 \ 6 \\ \hline 8 \ 4 \end{array} \quad \begin{array}{r} 3 \\ 2 \ 5 \\ \times 1 \ 7 \\ \hline 7 \ 5 \end{array} \quad \begin{array}{r} 3 \ 4 \\ \times 2 \ 3 \\ \hline 1 \ 2 \\ 9 \\ 8 \\ + \quad 6 \\ \hline 3 \ 5 \end{array} \quad \begin{array}{r} 2 \ 4 \\ \times 1 \ 6 \\ \hline 2 \ 4 \\ 1 \ 2 \\ 4 \\ + \quad 1 \ 2 \\ \hline 5 \ 2 \end{array} \quad \begin{array}{r} 3 \ 3 \\ \times \quad 6 \\ \hline 1, \ 8 \ 1 \ 8 \end{array}$$

The errors above are examples of those made by students who later benefited from this program. The errors show that students lack a sense of numbers, that multi-digit numbers are not just separate numerals, but each one has a different value (47 is 4 tens and 2 ones rather than a 4 and 2). Students who have attempted to memorize steps to the algorithm without a sense of numbers engage in various type of error patterns such as those above. The purpose of this program is to build students' sense of numbers and understanding of the multiplication operation. In addition, the program is about understanding the operation in the context of real-life situations. Therefore, each lesson presents computation problems with words that build into word problems and finally progress to differentiation between addition, subtraction, and multiplication problems. This allows students to engage in mathematical practices.

This manual may be appropriate for elementary students who struggle in learning the partial products algorithm as part of meeting grade-level standards. It may also be appropriate for older students who struggle in remembering and executing the shortened steps associated with the standard algorithm.

Results of Field Tests

The first author of this manual field-tested each of the lessons with elementary school students with disabilities as well as with students at-risk for failure. The first study used a single case design and involved three elementary students with high incidence disabilities in fourth and fifth grade. The students attended a high need school in a rural area (50% of students received

free or reduced lunch). Their special education teacher provided instruction during the students' regularly scheduled resource period. All three students demonstrated mastery of multiplication using partial products. Students' baseline accuracy scores were 0% and their accuracy scores were 100% after instruction. Their fluency scores improved from 5-10 correct digits to 35-100 correct digits.

Another field test by the first author of this manual included 15 fourth- and fifth-grade students without disabilities. The students were English language learners who attended an urban high-need school (100% of students received free or reduced lunch). General education teachers provided instruction during a 30-minute intervention period for students identified as at-risk for failure in mathematics. Prior to instruction, students' accuracy scores ranged between 0%-40%, with an average of 18%. After instruction, students' accuracy scores ranged between 50-100% with an average of 77%. Each of the students gained at least 40% between the pretest and posttest.

Prerequisite Skills

In order to benefit from instruction using this manual, students should have mastered basic multiplication facts (zero through five) plus addition and subtraction involving regrouping. If students have not mastered these skills, instruction should be provided prior to taking the pretest. Field-testing has shown that deficits in these areas will significantly interfere with student learning. The authors intentionally developed the problems in this manual so that students who have not mastered facts involving larger numbers such as 8×7 or 6×9 can be successful. Teachers may choose to use the *Strategic Math Series* to remediate deficits in the aforementioned areas. These include *Addition Facts 0 to 9* (Miller & Mercer, 1991a), *Subtraction Facts 0 to 9* (Miller & Mercer, 1991b), *Multiplication Facts* (Miller & Mercer, 1993), *Addition With Regrouping* (Miller, Kaffar, & Mercer, 2011), and *Subtraction With Regrouping* (Miller, Kaffar, & Mercer, 2011).

The Instructional Sequence

This manual included into eight instructional phases that include 18 lessons in which students compute problems and solve word problems. Instruction in word problems involves discrimination between operations to ensure understanding of multiplication. Lessons build upon each other, increasing in complexity as students make progress through the program. If students miss a lesson or perform poorly (less than mastery – see table below), it is important that they receive instruction and remediation rather than moving ahead to keep pace.

Phase	Purpose	Lessons	Mastery Criteria
Phase 1	Pretest	Pretest Lesson	80%
Phase 2	Teach at the concrete level	Lessons 1-4	100% 2/2 independent practice items
Phase 3	Teach at the representational level	Lessons 5-8	100% 2/2 independent practice items
Phase 4	Teach RENAME Strategy	Lesson 9	100% accuracy

Phase 5	Teach multiplication at the abstract level	Lesson 10	80% 4/5 independent practice items
Phase 6	Teach FAST RENAME Strategy for solving word problems	Lesson 11	100% accuracy
Phase 7	Teach computation and problem solving at the abstract level, differentiating between operations within word problems. Teacher guidance fades.	Lessons 12-14	80% 4/5 independent practice items
		Lesson 15-18	88% 7/8 independent practice items
Phase 8	Posttest	Posttest Lesson	
