

+ -× ÷



The Professional Development Research Institute Joseph R. Pearson Hall 1122 West Campus Road, Room 708 Lawrence, KS 66045-3101 785.864.0626 • 785.864.5728 (fax) simpd@ku.edu • http://sim.ku.edu

SIM Strategic Math Series

The Strategic Math Series is a collection of evidence-based teaching tools focused on teaching basic math facts and operations to students of any age.

Each book in The Strategic Math Series is based on the **"concrete-representational-abstract" (CRA)** method of instruction, an evidence-based practice for mathematics. In this approach, students first learn to understand a math concept by using concrete objects, then move to drawings or pictures like dots or tallies that represent a certain number of objects, and ultimately work with numbers alone, turning all students into active problem solvers. Each manual in the series includes instructional materials that apply the separated CRA sequence and the integrated CRA sequence. Within each level of instruction, students also learn to solve "word problems" using a simple strategy.

The series can be used as a supplementary curriculum for elementary students learning single-digit operations and regrouping concepts and algorithms, or as an intervention program for older students.

⁶ When I first started teaching I quickly realized that I didn't have the tools/skills to help the varying needs of all my students. SIM was my missing tool! **99**



sim.ku.edu



Strategic Math Series: Place Value

Place Value: Discovering Tens and Ones helps students learn place value to ten. The first three lessons identify place using manipulatives. The next set of lessons identify place value using drawings and pictures. Students learn a mnemonic strategy (FIND) to assist them in finding place value by identifying ones and tens places and inserting column lines. The final set of lessons use the FIND strategy and apply it to two-digit addition and subtraction that does not include regrouping.

Recommended for Students who:

- are developing conceptual understanding of place value to ten.
- need hands-on and visual models to observe the operation in action.
- need explicit instruction and repeated practice to develop fluency.

Addresses this standard:

 Grade Two - Understand that the two digits of a two-digit number represent amounts of tens and ones

In studies, students' scores rose from a pretest mean of 20% to a posttest mean of 88%. Their mean scores on place value tests were posttest--88%; maintenance test--97%; and retention test--96%.

Strategic Math Series: Addition



Addition Facts 0 to 9 helps students learn the addition operation of single digit numbers with sums to nine. Students learn a mnemonic strategy (DRAW to assist them in solving equations and avoid common errors by attending to the operational symbol and addends. The final set of lessons uses the DRAW strategy to build fluency using games.

Recommended for Students who:

- are developing conceptual understanding of the addition operation.
- need hands-on and visual models to observe the operation in action.
- need explicit instruction and repeated practice to develop fluency.

Addresses these standards:

- Kindergarten Understand addition as putting together and adding to, represent addition with objects, mental images, and drawings, solve word problems with objects and drawings
- Grade One Add within 20 (partially addressed)
- Grade Two: Know from memory all sums to 20 (partially addressed)

In a study, students' computation scores rose from a pretest mean of 16% to a posttest mean of 98%. The speed at which students were able to make their calculations increased by 400%.

Professional Learning

KUCRL is committed to finding solutions to educational challenges and placing our research findings into the hands of practitioners, students, and researchers in the field. Our expansive network of dedicated professionals — the SIM International Professional Development Network — shares our values and goals for delivering high-quality professional learning with a partnership approach to educators around the world. These experts offer professional development, instructional coaching, and technical assistance to establish the necessary infrastructure support for educators to implement evidence-based practices.

Addition Facts 10 to 18 helps students learn the addition operation of two digit numbers with sums ten to eighteen. The first three lessons solve equations using manipulatives. The next set of lessons solve equations using drawing and pictures. Students learn a mnemonic strategy (DRAW) to assist them in solving equations by avoiding common errors, attending to the operational symbol and addends. Students learn a mnemonic strategy (FAST DRAW) to solve equations and word problems. The final set of lessons uses the DRAW strategy and builds fluency using games.

Recommended for Students who:

- are developing conceptual understanding of the addition operation and basic place value concepts.
- need hands-on and visual models to observe numbers grouped by tens and the addition operation in action.
- need explicit instruction and repeated practice to develop fluency.

Addresses these standards:

- Grade One Add within 20 (partially addressed), understand place value to ten.
- Grade Two: Know from memory all sums of all two digit numbers (partially addressed to 20)

In a study, students word problem performance rose from a pretest mean of 42% to a posttest mean of 87%. A second group of students who scored 80% or higher on a computation pretest but below mastery on the word problem pretest increased their word problem scores from a pretest mean of 41% to a posttest mean of 91%.

Addition With Regrouping helps students learn the standard algorithm for addition with sums within 1000. Students learn a mnemonic strategy (RENAME) to assist them in using each step of the standard algorithm. The final set of lessons uses the RENAME strategy and builds fluency using games.

Recommended for Students who:

- already have conceptual understanding of simple addition.
- struggle with place value concepts beyond labeling.
- need explicit instruction and repeated practice to develop fluency.

Addresses these standards:

- Grade Two: Understand that the three digits of a threedigit number represent amounts of hundreds, tens, and ones. Add within 1000, using concrete models or drawings and strategies based on place value and properties of operations.
- Grade Three: Fluency within 1000 using strategies and algorithms based on place value and properties of operations.

Addition Within 20 helps students learn the addition operation, its relation to subtraction, the meaning of the equal sign, initial place value concepts, problem solving, and mental strategies for solving addition equations.

- The first set of lessons solve equations using manipulatives, number lines, and drawings.
- The next set of lessons solve equations using strategies such as counting-on and using a known fact with number lines, drawings, and pictures.
- Students learn a mnemonic strategy (FACTS) to assist them in solving equations by avoiding common errors (attending to the operational symbol and addends) and using mental strategies to solve equations.
- The final set of lessons uses the FACTS strategy and builds fluency using games.

 Recommended for Students who: are developing conceptual understanding of the addition operation and basic place value concepts. need hands-on and visual models to observe numbers grouped by tens and the addition operation in action. are developing understanding of mathematical symbols and associated vocabulary. need explicit instruction and repeated practice to develop fluency. subtract within 20 using mental strategies. 	 Addresses these standards: Grade One - Apply the commutative and associative properties of addition, understand the meaning of the equation sign, add within 20 demonstrating fluency, use strategies such as counting on and making tens; understand that the two digits of a two-digit number represent amounts of tens and ones; Grade Two: Fluently add and subtract within 20 using mental strategies.

Students in one study improved their scores on an assessment of number concepts and addition from 6-67% scores to scores of 100%.

Strategic Math Series: Subtraction



Subtraction Facts 0 to 9 helps students learn the subtraction operation of single digit numbers from zero to nine. The first three lessons solve equations using manipulatives. The next set of lessons solve equations using drawings and pictures. Students learn a mnemonic strategy (DRAW) to assist them in solving equation by avoiding common errors, attending to the operational symbol and the numbers. The final set of lessons uses the DRAW strategy and builds fluency using games.

Recommended for Students who:

- are developing conceptual understanding of the subtraction operation.
- need hands-on and visual models to observe the operation in action.
- need explicit instruction and repeated practice to develop fluency.

Addresses these standards:

- Kindergarten Understand subtraction as taking apart or taking from, represent subtraction with objects, mental images, and drawings, solve word problems with objects and drawings
- Grade One Subtract within 20 (partially addressed).

In studies, students with learning disabilities increased their ability to compute subtraction facts by 66% and the speed at which they were able to make these calculations by 230%. Their mean posttest scores: Computation - 97%, Word problems without extraneous Information - 100%, Word problems with extraneous information - 92% and Creation of word Problems - 82%.

Subtraction Facts 10 to 18 helps students learn the subtraction and initial place value concepts. The first three lessons solve equations using manipulatives. The next set of lessons solve equations using drawings and pictures. Students learn a mnemonic strategy (DRAW) to assist them in solving equation by avoiding common errors, attending to the operational symbol and the numbers. Students learn a mnemonic strategy (FAST DRAW) to solve equations and word problems. The final set of lessons uses the DRAW strategy and builds fluency using games.

Recommended for Students who: are developing conceptual understanding of the subtraction operation. need hands-on and visual models to observe numbers grouped by tens and the addition operation in action. need explicit instruction and repeated practice to develop fluency.

In a study, students computation scores rose from a pretest mean of 15% to a posttest mean of 98% and their speed in making these calculations increased 110%.

Subtraction with Regrouping helps students use strategies based on place value, specifically using the standard algorithm for subtraction of numbers within 1000. The first set of lessons solve equations using manipulatives. The next set of lessons solve equations using drawings and pictures. Students learn a mnemonic strategy (RENAME) to assist them in using each step of the algorithm. The final set of lessons uses the RENAME strategy and builds fluency using games.

 Recommended for Students who: are developing conceptual understanding of multi-digit subtraction. struggle with place value concepts and their application to operations and are learning multi-digit subtraction. need explicit instruction and repeated practice to develop fluency. 	 Addresses this standard: Grade Three - Fluently subtract within 1000 using strategies and algorithms based on place value and properties of operation.
Research studies demonstrated that student with disabilities and students who	preceived MTSS intervention performed significantly better and mastered the concept

with improvements in accuracy (0% to 100%) and fluency (1 correct digit to 30 correct digits).

Strategic Math Series: Multiplication



Multiplication Facts 0 to 81 helps students learn the multiplication operation using single digit numbers. The first three lessons solve equations using manipulatives. The next set of lessons solve equations using drawings and pictures. Students learn a mnemonic strategy (DRAW) to assist them in solving equations by avoiding common errors by attending to the operational symbol and the multipliers. The final set of lessons uses the DRAW strategy and builds fluency using games.

Recommended for Students who:

- are developing conceptual understanding of the multiplication operation.
- need hands-on and visual models to observe the operation in action.
- need explicit instruction and repeated practice to develop fluency.

Addresses this standard:

 Grade Three - Interpret products of whole numbers, e.g., interpret 3 x 7 as the total number of objects in 3 groups of 7 objects each; apply the commutative property of multiplication.

In studies, computation scores of students with learning disabilities rose from a pretest mean of 45% to a posttest mean of 92% and their speed in making theses calculations increased 400%. Mean posttest scores: Word problems without extraneous information - 100%, Word problems with extraneous information - 92% and Creation of word problems - 71%.

Multiplication With Regrouping: Partial Products helps students learn to use strategies based on place value, specifically the partial products algorithm for multiplication of two-digit numbers and solve word problems that include multiplication. Students learn to use estimation to evaluate the reasonableness of their work.

- Word problems are presented in each lesson, and after students master the multiplication algorithm, they solve word
 problems that require either addition, subtraction or multiplication so that they engage in mathematical practices and
 differentiate between operations.
- The first set of lessons solve equations using manipulatives.
- The next set of lessons solve equations using drawings and pictures.
- Students learn a mnemonic strategy (RENAME) to assist them in using each step of the partial products algorithm.
- The final set of lessons uses the RENAME strategy and builds fluency using games.

Recommended for Students who:

- already have conceptual understanding of simple multiplication. They do not need to have memorized all multiplication facts.
- are developing conceptual understanding of multi-digit multiplication
- struggle with understanding place value beyond labeling.
- need support to differentiate between additive and multiplicative reasoning when solving word problems.
- need associated vocabulary support.
- need explicit instruction and repeated practice to develop fluency.

Addresses these standards:

- This manual is aligned with current mathematics standards including mathematical practices for problem solving.
- Grade Four: Multiply two two-digit numbers, use strategies based on place value and the properties of operations.
- Grade Five: Recognize that in a multi-digit number, a digit in one place represents 10 times such as it represents in the place to its right and 1/10 of what it represents in the place to its left, explain patterns in the number of zeros of the product when multiplying a number by powers of 10.

Research studies demonstrated that students with and without disabilities receiving MTSS intervention increased their accuracy (0% to 100%) and fluency (9 correct digits to 60 correct digits) after instruction.

Multiplication with Regrouping: Standard Algorithm helps students learn to use strategies based on place value,

specifically the standard algorithm for multiplication of two-digit numbers and solve word problems that include multiplication.

- Word problems are presented in each lesson, and after students master the multiplication algorithm, they solve word
 problems that require either addition, subtraction or multiplication so that they engage in mathematical practices and
 differentiate between operations.
- The first set of lessons solve equations using manipulatives and drawings.
- The next set of lessons solve equations using drawings and pictures.
- Students learn a mnemonic strategy (RENAME) to assist them in using each step of the standard algorithm.
- The final set of lessons uses the RENAME strategy and builds fluency using games.

Recommended for Students who:

- already have conceptual understanding of simple multiplication. They do not need to have memorized all multiplication facts.
- are developing conceptual understanding of multi-digit multiplication
- struggle with place value concepts and their application to operations.
- need support to differentiate between additive and multiplicative reasoning when solving word problems.
- need associated vocabulary support.
- need explicit instruction and repeated practice to develop fluency.

Addresses these standards:

- This manual is aligned with current mathematics standards including mathematical practices for problem solving.
- **Grade Four**: Multiply two two-digit numbers, use strategies based on place value and the properties of operations.
- Grade Five: Recognize that in a multi-digit number, a digit in one place represents 10 times such as it represents in the place to its right and 1/10 of what it represents in the place to its left, explain patterns in the number of zeros of the product when multiplying a number by powers of 10

Research studies demonstrated that students with and without disabilities receiving MTSS intervention increased their accuracy (0% to 100%) and fluency (0 correct digits to 30 correct digits) after instruction.



Strategic Math Series: Division

Division Facts 0 to 81 helps students learn to use the division operation using two digit numbers. The first three lessons solve equations using manipulatives. The next set of lessons solve equations using drawings and pictures. Students learn a mnemonic strategy (DRAW) to assist them solving equations by avoiding common errors by attending to the operational symbol and the numbers. The final set of lessons uses the DRAW strategy and builds fluency using games.

Recommended for Students who:

- are developing conceptual understanding of the division operation.
- need hands-on and visual models to observe the operation in action.
- need explicit instruction and repeated practice to develop fluency.

Addresses this standard:

• Grade Three - Interpret whole-number quotients of whole numbers, e.g., interpret 48 + 8 as the number of objects in each share when 48 objects are partitioned equally into 8 shares, or as a number of shares when 48 objects are partitioned into equal shares of 8 objects each.

In studies, students' computation scores rose from a pretest mean of 45% to a posttest mean of 92%. Their speed in making these calculations increased 87%.



Institute for Strategic Mathematics Interventions

https://kucrl.ku.edu/ismi

Co-directed by Strategic Math Series authors Margaret Flores and Bradley Kaffar, the goals of the Institute for Strategic Mathematics Interventions at the University of Kansas Center for Research on Learning are to:

- develop and rigorously evaluate mathematics interventions that focus on conceptual knowledge and align with current mathematics standards
- engage in research and development of the Strategic Math Series
- support educators in the implementation of effective and efficient mathematics instruction to engage all students and improve learning.